

Theoretical Foundations: Why Fiscal Sustainability Imposes a Ceiling on Optimal Monetary Policy

A preemptive response to technical criticisms of the central argument — with references to the formal models that support it

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Context note: This text accompanies the article *"Brazilian Monetary Policy Is Running Out of Room"* and is intended for readers with a technical background in macroeconomics. Its purpose is to anticipate objections and show that the central argument — that the fiscal constraint compresses the optimal Selic rate — has formal support in the literature on optimal monetary policy, the fiscal theory of the price level, and dynamic general equilibrium.

01 // The Ramsey Problem: What Optimal Monetary Policy Is

The starting point is the canonical optimal monetary policy problem in the New Keynesian model, formulated by Woodford (2003) and Benigno & Woodford (2004). The central bank solves:

CENTRAL BANK LOSS FUNCTION

$$\min E_0 \sum \beta^t [\lambda_\pi (\pi_t - \pi^*)^2 + \lambda_x x_t^2]$$

π_t = inflation, π^* = target, x_t = output gap, λ_π and λ_x = relative weights

subject to the New Keynesian Phillips Curve and the Dynamic IS Equation:

PHILLIPS CURVE (NK)

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

DYNAMIC IS

$$x_t = E_t x_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1} - r_t^*)$$

The solution to this problem without a fiscal constraint yields the optimal interest rate path $\{i_t\}$ that minimizes the joint cost of inflation deviation and output volatility. The first-order condition for i_t is:

FOC — UNCONSTRAINED PROBLEM

$$\lambda_\pi \pi_t + \varphi_t \kappa = 0$$

φ_t = Lagrange multiplier on the Phillips Curve

RESULT

The optimal interest rate path equalizes the marginal cost of inflation ($\lambda_\pi \pi_t$) with the marginal benefit of narrowing the gap via the Phillips Curve ($\phi_t \kappa$). This is the framework the Copom itself implicitly uses when it refers to "alternative paths compatible with the smoothing of macroeconomic aggregates" — language that maps directly onto minimizing a loss function with a penalty on output variance.

EXPECTED OBJECTION

"The central bank does not use a formal loss function. The reference to the Copom minutes is the analyst's projection onto the text."

The objection is partly correct — the Copom does not publish λ_π and λ_x . But the language of the May 2026 minutes is structurally compatible with this framework: by stating that "alternative paths ensuring inflation's convergence to target are compatible with smoothing in the variation of macroeconomic aggregates," the Committee is describing the solution set of a loss function penalizing GDP variance. This interpretation is not projection — it is a translation of institutional central-bank-speak into its formal equivalent. (See also: *Flexible Average Inflation Targeting*, Fed, 2020 — same implicit structure.)

02 // The Fiscal Constraint: The Transversality Condition as a Constraint on the Ramsey Problem

When the government's budget constraint is added as an explicit constraint to the Ramsey problem, the problem changes. The central reference here is **Benigno & Woodford (2006)** *"Optimal Taxation in an RBC Model"* and **Schmitt-Grohé & Uribe (2004)** *"Optimal Fiscal and Monetary Policy Under Sticky Prices"*.

The dynamics of debt as a share of GDP ($b = B/Y$, where B is nominal debt and Y is nominal output) are:

DEBT-TO-GDP ACCUMULATION

$$b_t = R_t \cdot b_{t-1} - s_t$$

$R_t = (1 + i_t)/(1 + g_t)$ = debt growth factor adjusted for GDP growth; s_t = primary surplus/GDP

The transversality condition requires that this path not be explosive:

TRANSVERSALITY CONDITION (TVC)

$$\lim_{T \rightarrow \infty} \beta^T \cdot \mu_T \cdot b_T = 0$$

μ_t = Lagrange multiplier on the debt-accumulation constraint

The intuition behind the TVC is precise: among all paths that satisfy the Euler equations (candidates for the optimum), only the one in which the present value of debt converges to zero is truly optimal — because any other implies the agent is accumulating wealth (or debt) whose present value is not zero, which contradicts optimality. **In the fiscal problem, the TVC is the formal prohibition of a sovereign Ponzi scheme:** the government cannot roll over debt indefinitely without fiscal adjustment.

EXPECTED OBJECTION

"The TVC is an equilibrium condition, not an active constraint on the Selic rate. The central bank does not maximize taking debt into account."

This is true under the regime of monetary dominance (Leeper, 1991) — where fiscal policy is passive and adjusts to guarantee the TVC regardless of the interest rate. The article's argument does not deny this regime as a reference point. What it argues is that when fiscal policy is *active* — when the surplus s_t does not adjust automatically — the TVC is no longer satisfied

automatically and instead imposes an effective constraint on the set of sustainable i_t paths. This is precisely Leeper's (1991) classification: Brazil is moving along the monetary-dominance spectrum toward a fiscal-dominance regime — it has not arrived there yet, but the margin is shrinking.

03 // What Changes in the FOC When the Fiscal Constraint Is Active

When $\mu_t > 0$ (active fiscal constraint, not automatically satisfied), the first-order condition for i_t in the Ramsey problem with a fiscal constraint becomes:

FOC — PROBLEM WITH FISCAL CONSTRAINT (BENIGNO-WOODFORD 2006)

$$\lambda_t \pi_t + \phi_t \kappa + \mu_t \cdot (\partial R_t / \partial i_t) \cdot b_{t-1} = 0$$

$\partial R_t / \partial i_t = 1/(1 + g_t) > 0$ — a rate hike increases the debt growth factor

The third term is the new element — and it is the core of the argument. Since $\partial R_t / \partial i_t > 0$ and $b_{t-1} > 0$, this term is positive. For the equality to hold, i_t must be **lower** than in the unconstrained problem. Formally:

RESULT

The larger the debt stock b_{t-1} and the more active the fiscal multiplier μ_t , the further the optimal interest rate path is pushed below the level an unconstrained central bank would choose. The optimal Selic rate is a decreasing function of b and of μ — and both b and μ are rising in Brazil in 2026.

EXPECTED OBJECTION

"With gross debt at 82.7% of GDP, Brazil is still far from any fiscal-dominance threshold. The multiplier μ is small or zero."

The argument does not require μ_t to be large — it requires μ_t to be **rising**. The central point is not that fiscal dominance is already in effect, but that the $(r - g)$ spread of 600 basis points, combined with a quasi-fiscal policy that adds 1.5%–2.0% of GDP in demand stimulus with no offsetting surplus, is increasing b_{t-1} and eroding the credibility of the s_t path — which raises μ_t over time, even starting from a contained level. The mechanism is gradual, not binary.

The criticism that "82.7% of GDP is not high enough for fiscal dominance" conflates two distinct regimes: active fiscal dominance (rupture) and a growing approach toward the implicit ceiling (what the article claims). The argument is about the derivative, not the level.

04 // The Fiscal Theory of the Price Level: When Tightening Becomes Pro-Inflationary

The limiting case — in which additional tightening becomes pro-inflationary — is formalized in the **Fiscal Theory of the Price Level (FTPL)**, developed by Woodford (1994, 1995), Sims (1994), Cochrane (1998, 2023), and Leeper (1991), and applied to the case of fiscal pressure in Sims (2011) *"Stepping on a Rake"*.

In the FTPL, the government's intertemporal budget constraint determines the price level:

FTPL — PRICE LEVEL DETERMINATION

$$B_{t-1} / P_t = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$$

B_{t-1} = nominal debt stock (fixed at time t); P_t = price level; s_t = real primary surplus

The transmission mechanism is direct: if the market revises downward its expectation of the future surplus path $E_t \sum s_{t+j}$ — without B_{t-1} adjusting (it is a given nominal stock) — the only equilibrating adjustment is a rise in P_t . Inflation is not

caused by an excess of money, but by an excess of debt relative to expected surpluses.

EXPECTED OBJECTION

"The FTPL is controversial. Cochrane and Sims have strong critics — Buiter, McCallum, Bassetto. Using it as support weakens the argument."

The objection deserves respect. The FTPL was indeed contested in the literature of the 1990s and 2000s, principally in Buiter's (2002) argument about the distinction between genuine budget constraints and equilibrium conditions, and in McCallum's critique of the TVC's status as an equilibrium-selection condition. The theory's current standing, however, is more established than that critique suggests — Cochrane's (2023) book-length formalization responds point by point to the canonical objections, and the theory has gained growing acceptance alongside post-2020 empirical experience in the US (post-fiscal, not monetary, inflation). For the article's argument, it is worth noting that the FTPL is not used as the sole support — it is presented as the formalization of the limiting case. The main case (Section 03) rests on Benigno-Woodford (2006) and Schmitt-Grohé & Uribe (2004), which are established literature with no controversy over their status.

The Sims (2011) result the article uses is specific: when μ_t is sufficiently high, a rise in i_t increases $R_t \cdot b_{t-1}$ without the market believing in compensation via s_t , forcing P_t upward as the mechanism restoring the TVC. In this regime:

PARADOX — "STEPPING ON A RAKE" (SIMS 2011)

$$\uparrow i_t \rightarrow \uparrow R_t b_{t-1} \rightarrow \downarrow E_t \sum s_{t+j} \text{ (credibility)} \rightarrow \uparrow P_t$$

The equilibrating adjustment shifts from the fiscal channel (adjustment of s_t) to the price channel (adjustment of P_t) — monetary tightening becomes pro-inflationary

RESULT

The article does not claim that Brazil is in this regime today. It claims that Brazil is approaching the point where this mechanism becomes relevant — and that the current quasi-fiscal trajectory is moving the economy in that direction, not the opposite.

05 // Summary: How to Respond to a Technical Critic in Three Steps

STEP	ARGUMENT	FORMAL REFERENCE
01	Optimal monetary policy minimizes a loss function over inflation and output variance. The Copom minutes from May/26 use language compatible with this framework without formally naming it.	Woodford (2003); Benigno & Woodford (2004)
02	When fiscal policy is active — the surplus does not adjust automatically — the TVC imposes an effective constraint on the optimal interest rate path. The FOC gains a third term proportional to the debt stock and the fiscal multiplier, compressing the optimal Selic rate below the level that would prevail under sound public finances.	Leeper (1991); Benigno & Woodford (2006); Schmitt-Grohé & Uribe (2004)
03	In the limit where the fiscal constraint approaches being binding, additional tightening can become pro-inflationary via the FTPL channel — because the market anticipates that the TVC can only be satisfied through a price-level adjustment, not a fiscal adjustment. Brazil is not in this regime today, but the trajectory points in that direction.	Sims (2011); Cochrane (2023); Woodford (1994)

The article positions itself between step 2 and step 3 of this summary. The central argument — that the fiscal constraint significantly compresses the optimal Selic rate without fiscal dominance being formally in effect — is formalized by **Benigno & Woodford (2006)** and **Schmitt-Grohé & Uribe (2004)**, established literature that is not controversial regarding its theoretical status.

The reference to the FTPL is used to illuminate the **risk mechanism** — what happens if the current trajectory is not corrected — not as the argument's primary support. This distinction matters for responding to critics who might use the controversies surrounding the FTPL to discredit the entire argument: the core of the argument does not depend on the FTPL being accepted, even though the FTPL offers the clearest formalization of the limiting case.

Finally: the idea that the Brazil studied here does not fit the standard monetary-dominance model is empirically corroborated by the experience of the 1980s and 1990s, in which the concentration of debt in overnight instruments was simultaneously a symptom and an accelerator of the inflationary spiral. The theory formalizes what Brazilian history has already demonstrated in practice.

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