ASSET PRICING · NTN-B · TERM PREMIUM · EMPIRICAL EXTENSION

The Market Isn't Wrong: Quantifying Brazil's Fiscal Risk Premium

An asset-pricing argument is only as good as its numbers. Applying a no-arbitrage term structure model to Brazil's real yield curve, the fiscal risk premium is now measurable, statistically robust to a global control — and smaller than the raw numbers alone would suggest.

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Institutional explanations for Brazil's elevated NTN-B yields — saturation of regular buyers, shrinking multimarket funds, regulation — are real technical factors, but insufficient to explain real yields above 8% and breakevens above 5.5%. The right explanation lives in asset-pricing theory, not market microstructure. This note builds that theoretical case from first principles, then makes it empirical.

Part I: The Theoretical Case

Every asset prices as:

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P = E[M · X]
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where X is the payoff and M is the stochastic discount factor. M generalizes the familiar idea of a discount rate: just as interest rates discount a payoff by time, M discounts the payoff also by the state of nature it occurs in — each future scenario carries its own "price of time and risk." M is high in bad states (low consumption, high marginal utility) and low in good states, reflecting that an extra real is worth more to the agent exactly when things go badly. An asset is worth little — and pays a lot — when its payoff covaries negatively with M: that is, when it pays badly exactly in the states where the agent most values an extra real.

The NTN-B has exactly that feature today. In bad fiscal states — spending indiscipline, deteriorating debt trajectory — three things worsen simultaneously, all in high-M states:

> Duration — the real yield curve rises, penalizing price via a higher discount rate.

> Credit — the perceived probability of reprofiling or financial repression rises.

> Liquidity — dealers cut book, the bid-ask widens, and the ability to exit worsens exactly when the agent most needs liquidity. Liquidity is not an independent factor — it covaries with the same bad states that worsen duration and credit, and that covariance is priced too.

What changed recently was not the representative agent's risk aversion — it was the probability assigned to these bad states. With the election approaching, the market appears to be pricing a higher probability of continuity of the current fiscal policy, and with it, a higher probability of the spending-indiscipline and debt-deterioration states. In other words, the channel is P, not M: the probability distribution over states of nature shifted, increasing the weight on scenarios where the NTN-B pays badly. That alone is enough to raise the required risk premium, even with no change in the agent's risk aversion.

The same logic extends to the breakeven. In the same fiscal-stress states, inflation also tends

to be higher — whether through the financial-repression channel or because fiscal dominance itself resolves, ultimately, via inflation. Since inflation also covaries negatively with M, the breakeven carries its own risk premium, additional to the real premium:

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breakeven = expected inflation + inflation risk premium
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The market is not failing to price this correctly — it is doing exactly what asset-pricing theory predicts: discounting more heavily a payoff that worsens in the states that gained probability. The cause is fiscal and identifiable, and the measures to reverse this channel are well known: cutting tax expenditures, delinking the minimum wage, delinking health and education spending. Technically, the fix is simple. What's missing is political convergence around a credible fiscal adjustment.

Part II: The Empirical Extension

That argument is theoretical. This note makes it empirical, applying a no-arbitrage term structure methodology already developed and validated for Brazil's nominal (DI) curve to the real (NTN-B) curve, and extending the resulting term premium decomposition to the breakeven — with two rounds of honest methodological correction along the way, detailed as they occur below rather than smoothed over.

Methodology

The workhorse is the Adrian-Crump-Moench (2013) affine term structure model, with a Bauer-Rudebusch-Wu (2012) bootstrap bias correction on the persistence of the underlying state variables. In plain terms: the model summarizes the whole yield curve on any given day with a small number of statistical factors (level, slope, curvature — recovered via principal components), estimates how those factors evolve over time, and then uses a no-arbitrage restriction to split each observed yield into two pieces:

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observed yield  =  expected average short-rate path  +  term premium
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The first piece is what the yield would be if investors only cared about the expected path of short-term rates (the pure expectations hypothesis). The second is compensation for risk — the same mechanism described in the companion note's stochastic-discount-factor language, now given a number. The bootstrap correction matters because, in small samples, the raw estimate of factor persistence is biased downward, which materially understates the term premium; Bauer-Rudebusch-Wu correct this via a bootstrap re-estimation of the bias itself, rather than an ad hoc adjustment.

This is the identical estimation machinery already built and validated on Brazil's nominal DI curve in a separate note — same three-factor PCA state space, same bootstrap correction, same risk-price recovery. The only thing that changes here is which curve it is applied to.

Building the Real Curve from Individual Bonds

Brazil's official real-yield curve (ANBIMA's ETTJ) is refit daily from whichever bonds are trading, but its public data feed only retains roughly the last thirteen months — not enough for a term-premium model that needs to see multiple interest-rate cycles. Individual NTN-B bond tickers do have deep history, but each only covers its own life span (issuance to maturity), so no single instrument spans the full sample either.

The fix: pull every individual NTN-B bond's own yield history — 33 bonds in total, spanning November 2005 through August 2026 — and, for each trading day, fit a Nelson-Siegel-Svensson curve across whichever bonds were outstanding that day. This is mechanically what ANBIMA

itself does; the difference is that it is now reproducible and extendable to any maturity, including ones ANBIMA's own vertex grid doesn't cover.

The construction fit cleanly: 5,166 of 5,166 trading days fit successfully, with a mean pricing error of 7 basis points against the actual observed bond yields. Independently cross-checked against a separate, independently-built real curve (constant-maturity 1Y/2Y/5Y/10Y series built from ANBIMA official daily prices), the two curves agree to within a median 1-4bp with no systematic bias — reassuring, given the two were built by entirely different methods.

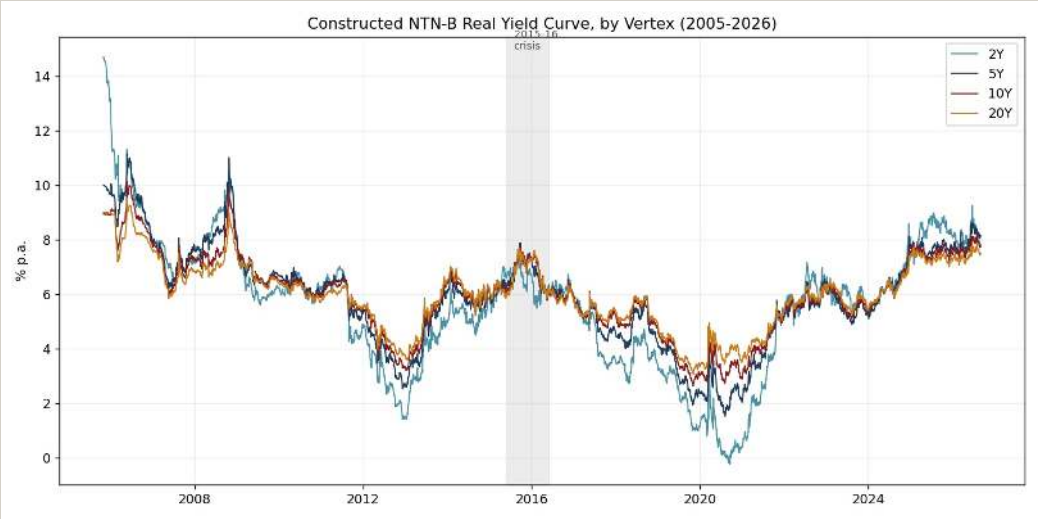

Figure 1. Real yield curve constructed from cross-sectional Nelson-Siegel-Svensson fits across all outstanding NTN-B bonds, 2005–2026. Every major Brazilian rate cycle is visible without being told where to look: the pre-2008 high-real-rate regime, the 2011-13 easing trough, the 2015-16 fiscal crisis, the pandemic collapse toward zero, and the current 2024-26 rise.

The Real Term Premium

Applying the ACM/BRW decomposition to the constructed real curve, targeting the 2Y, 5Y, 10Y and 20Y vertices, produces a term premium series spanning the full 2005-2026 sample. The Q-measure dynamics are stable throughout (largest eigenvalue modulus 0.996, no explosion), and the model refits the actual observed curve tightly (mean error near zero, standard deviation 0.09-0.17pp across vertices).

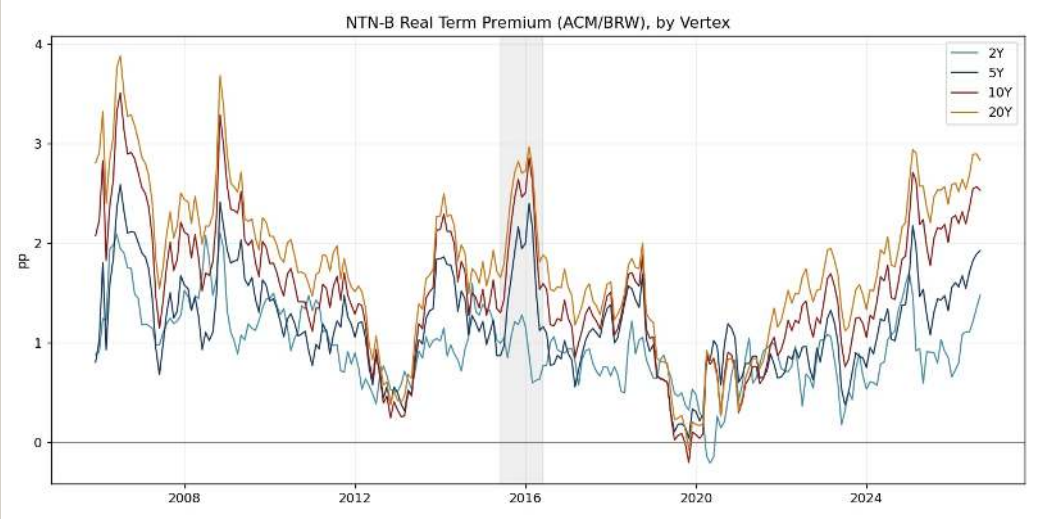

Figure 2. Real term premium, by vertex, 2005–2026. Shaded region marks the 2015-16 fiscal crisis.

The premium behaves exactly as the theory predicts: it rises with maturity (longer bonds carry more of the risk being priced), it spikes clearly during the 2015-16 crisis, compresses toward — and briefly through — zero during the pandemic's emergency easing, and has been rising steadily since 2022. As of end-August 2026:

VERTEX	REAL YIELD	EXPECTED PATH	TERM PREMIUM
2Y	7.76%	6.50%	1.48pp
5Y	8.16%	5.88%	1.92pp
10Y	7.78%	5.25%	2.53pp
20Y	7.60%	4.77%	2.83pp

At the 20-year point, the current term premium (2.83pp) is now approaching the 2015-16 crisis peak (2.97pp, January 2016) — the worst fiscal-risk episode in Brazil's modern rates history, at least on the raw, uncontrolled comparison. Whether that comparison survives once global term-premium co-movement is accounted for is the subject of a later section — and the answer is more qualified than the raw numbers alone suggest.

Risk pricing, or the technical factors this note opened by dismissing?

Part I dismissed institutional explanations — buyer saturation, shrinking multimarket funds, regulation — as insufficient on their own. That was asserted from theory, not tested. With an actual model in hand, it can be checked, at least indirectly: if these elevated premiums were mainly a technical dislocation — dealers pulling back, a temporary buyer-seller imbalance — rather than a coherent repricing of risk, the yield curve should get *messier* exactly when the premium spikes: harder to fit smoothly, more cross-sectional noise, evidence of segmented pricing at specific maturities.

It doesn't. The same cross-sectional fitting error used as the liquidity proxy in the breakeven decomposition below is a direct measure of exactly this — how well a single smooth curve explains all outstanding bonds on a given day. During both stress episodes, it sits at essentially normal levels: 0.071pp average during the 2015-16 crisis and 0.068pp during the current episode, against a 0.073pp full-sample average. Even the single worst month of each episode (January 2016, January 2025) shows fitting error well below the 90th percentile of the whole 2005–2026 sample. The curve prices smoothly and consistently throughout — including while yields are rising sharply.

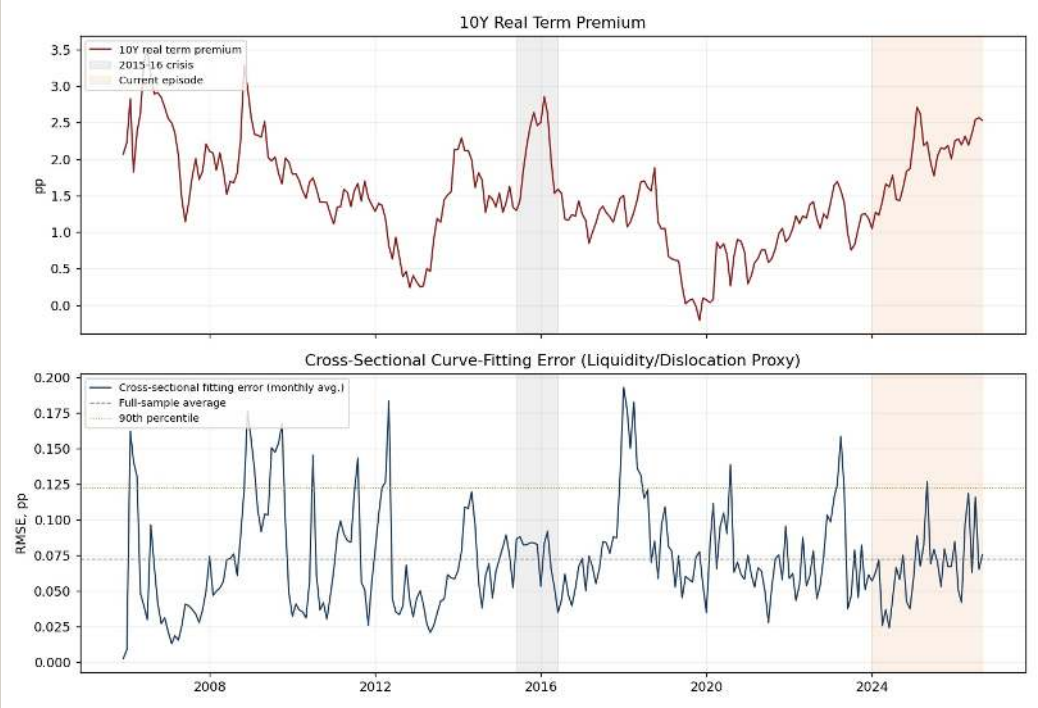

Figure 3. Top: 10Y real term premium. Bottom: cross-sectional NSS fitting error (RMSE against observed bond yields), a direct proxy for curve dislocation. The premium spikes in both shaded episodes; the fitting error does not.

Three further features point the same direction: the premium scales smoothly and monotonically with duration (2Y through 20Y, no kinks at specific maturities, which a

segmented-clientele or technical story would tend to produce); it spikes specifically at recognized fiscal-stress episodes rather than at moments associated with fund-flow events; and — even after the correction below cuts the estimate substantially — it carries a real, significant loading on the US real term premium, which a purely local technical dislocation would have no reason to do.

None of this is a clean, direct test — it doesn't use actual flow data (multimarket fund AUM, foreign NTN-B holdings, primary dealer positioning), which would be the natural way to settle the question properly and is flagged as a next step rather than attempted here. It's also worth being precise about what the model can and can't separate: an affine term structure model doesn't split the premium into "fundamental risk" and "technical" line items — anything priced and persistent, including a structural liquidity or balance-sheet constraint if one exists, gets folded into the same term premium this note reports. What the evidence above does support is narrower but still useful: whatever is driving the elevated premium, it is not showing up as the kind of cross-sectional pricing breakdown a temporary technical dislocation would produce. The curve is expensive and coherent, not dislocated.

Decomposing the Breakeven

The companion note's second empirical claim was that the breakeven (nominal minus real yield) should widen through the same channel: in the same fiscal-stress states that raise the real premium, expected inflation also tends to be higher, and that covariance with the discount factor should show up as its own, separate inflation risk premium. Testing this requires a nominal curve built with the same rigor as the real one — and, as an earlier pass through this analysis showed, it requires estimating the nominal and real curves *jointly*, not separately.

A methodological correction

An initial version of this decomposition fit the nominal (DI) and real (NTN-B) curves as two independent ACM models and simply subtracted one from the other. That produced an inflation risk premium that swung wildly (-2.9pp to +2.7pp at 2Y) and was negative on average — a result with no clean economic story and no real precedent in the Brazil-specific literature, which finds a premium that is small, smoothly increasing with maturity, and mostly positive (Vicente, 2021, BCB Working Paper 543). The fix, following the same joint estimation approach used for the US control below and by the closest published Brazil precedent (Kubudi & Vicente, BCB Working Paper 452), is to fit both curves *together*, sharing a consistent factor structure and anchored to realized IPCA — using the `RealACM` implementation of Abrahams, Adrian, Crump and Moench (2016), the same tool used for the US decomposition.

Brazil's DI futures — the instrument underlying the nominal curve — are only meaningfully liquid out to roughly 9-10 years; there is essentially no tradable nominal curve beyond that. The real curve, built from actual NTN-B bonds with maturities out to 2060, has no such constraint. This is a genuine data limitation, not a modeling choice: the breakeven decomposition below is therefore reported at 2Y, 5Y and 10Y only. The 20Y real term premium reported above stands on its own, with no nominal or breakeven counterpart available from this data source.

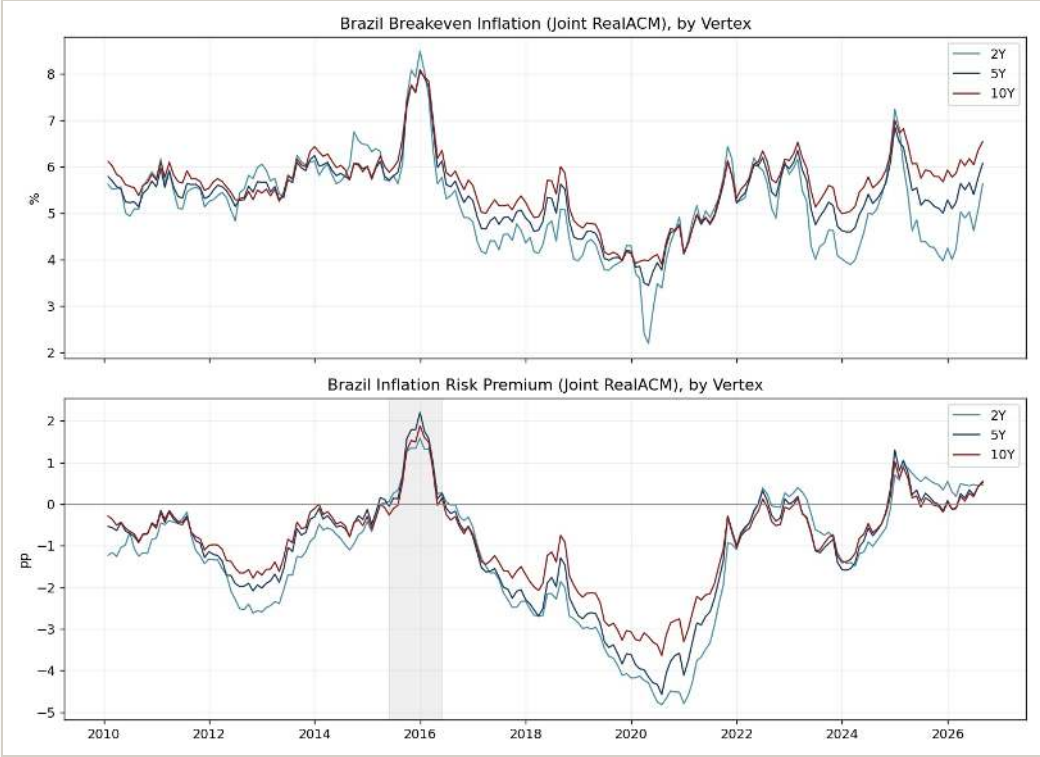

Figure 4. Top: breakeven inflation (joint RealACM). Bottom: inflation risk premium, by vertex, 2010–2026. Shaded region marks the 2015-16 fiscal crisis.

VERTEX	NOMINAL TP	REAL TP	BREAKEVEN	INFLATION RISK PREMIUM
2Y	1.09pp	0.62pp	5.63%	+0.47pp
5Y	2.63pp	2.09pp	6.07%	+0.55pp
10Y	3.70pp	3.18pp	6.54%	+0.52pp

With the joint model, the inflation risk premium is now well-behaved and economically legible across the full 2010–2026 sample. Two episodes anchor the read:

> 2015-16 fiscal crisis — the premium rises cleanly across all three vertices, peaking in December 2015 at +1.6pp (2Y), +2.2pp (5Y), +1.9pp (10Y) — the same crisis-sensitivity the real premium showed, now visible on the inflation side too.

> 2020-21 pandemic — the premium falls to its deepest point in the whole sample, -4.6pp at 5Y in July 2020. This is not a contradiction; it matches a pattern already documented in the international literature (Campbell, Sunderam and Viceira, 2016; ECB Working Paper 2033, 2017): during acute deflation scares, nominal bonds stop behaving as an inflation bet and start behaving as a deflation hedge, which shows up as a sharply negative inflation risk premium. Brazil's Selic fell to 2% during this period with real fears of a demand collapse — the mechanism fits.

The current reading — a small, positive premium of roughly 0.5pp across all three vertices — sits far below the 2015-16 crisis peak and close to the calm-period norm the Brazil-specific literature reports. Unlike the real term premium, the inflation side of this story is not currently signaling elevated fiscal-inflation stress.

The Global Control: Does Brazil's Story Survive?

Both of the results above are Brazil-only estimates. A skeptical reader's fair objection is that global term premia move together — a rise in the Brazilian real premium could partly reflect a global risk-appetite shift rather than anything specific to Brazilian fiscal risk. The companion research program's BRL note already addressed this for FX by explicitly testing a US term

premium control. The same discipline belongs here.

The US real term premium was estimated with the joint Treasury-TIPS decomposition of Abrahams, Adrian, Crump and Moench (2016) — the natural real-curve counterpart to the nominal ACM model, via the open-source `pyacm` implementation, fed with generic Treasury and TIPS yields (2Y–30Y) and a VIX-based liquidity control, following Andreasen, Christensen and Riddell (2021).

A second methodological correction: levels vs. differences

An initial pass at this control ran Brazil's real premium on the US real premium in levels, finding an implausibly strong relationship (R^2=0.55 at 10Y). Unit root tests (ADF, KPSS) on both series told a clear story before the regression should have been trusted: the US real term premium cannot be distinguished from a unit-root process by either test, and Brazil's own series is borderline. Two persistent, independently-trending series will produce an inflated R^2 in levels even with no genuine short-run linkage — precisely the problem the DI note's own US control section had already run into and fixed by switching to differences. The same fix applies here.

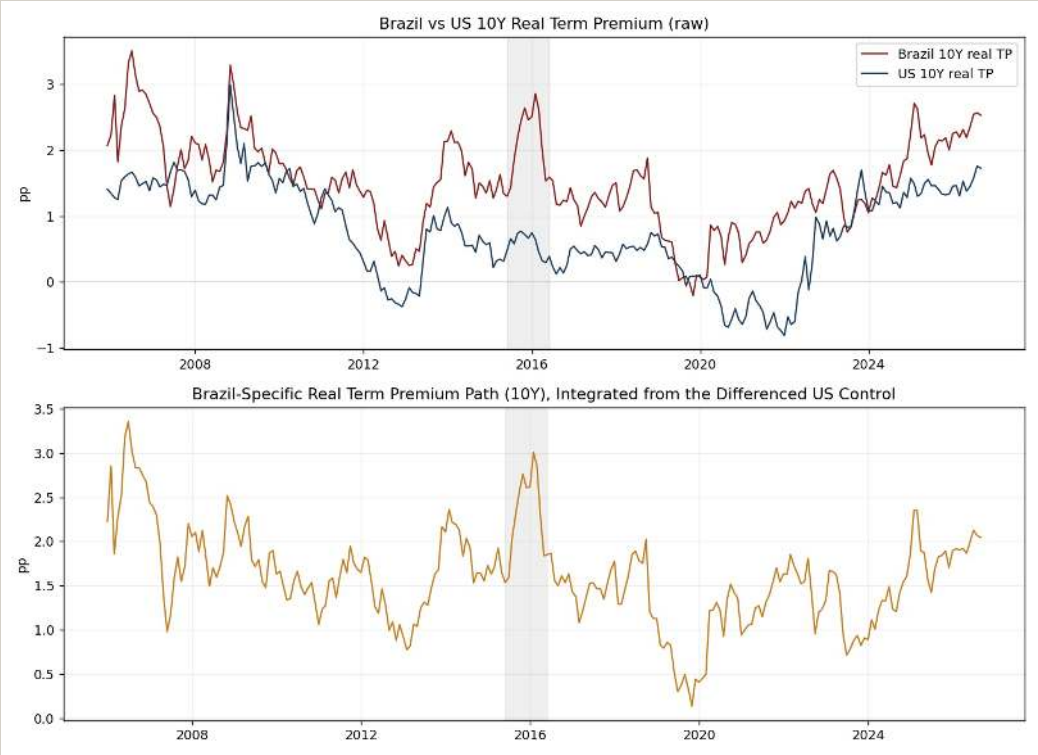

Figure 5. Top: Brazil vs US 10Y real term premium, raw. Bottom: the Brazil-specific path — reconstructed by cumulating the residuals of the (correctly specified) differenced regression of Brazil's premium on the US premium, i.e. the part of Brazil's level not explained by short-run global co-movement. Shaded region marks the 2015-16 fiscal crisis.

VERTEX	BETA ON ΔUS TP	T-STAT	R^2
5Y real TP	0.24	3.0	0.03
10Y real TP	0.44	5.4	0.11

The relationship is real — both betas remain statistically significant — but far smaller than the levels regression implied: R^2 falls from 0.55 to 0.11 at 10Y, roughly a fifth of the earlier estimate. Most of what looked like tight global co-movement was two series sharing a slow common drift over 2005–2026, not a strong contemporaneous link. The corrected reading is the opposite of the first-pass conclusion: the US explains only a modest share of Brazil's real premium, and the Brazil-specific component is even more dominant than a naive levels regression would suggest.

	RAW 10Y REAL TP	BRAZIL-SPECIFIC PATH
2015-16 crisis peak	2.97pp	3.01pp
Current (Aug 2026)	2.53pp	2.04pp

THE CORRECTED HEADLINE CLAIM

With the properly specified control, the current episode's Brazil-specific component (2.04pp) sits meaningfully below the 2015-16 peak (3.01pp) but not nearly as far below as the flawed levels-based estimate suggested (0.34pp vs 1.44pp). The honest summary: the current real term premium is elevated for both global and Brazil-specific reasons, the Brazil-specific part is real and substantial, and it is running somewhat, but not dramatically, below the worst acknowledged fiscal crisis in the modern sample.

The inflation risk premium's relationship to the US survives the same scrutiny less comfortably. Both series show similar non-stationarity concerns (Brazil's 10Y IRP: ADF p=0.36; US's: KPSS p=0.01), so the same differenced specification applies. In differences, Brazil's IRP still loads positively on the US IRP — the sign correction from the earlier methodological fix holds — but the relationship is markedly weaker than the levels regression suggested and, at 10Y, only marginally significant (5Y: β=0.28, t=2.2; 10Y: β=0.26, t=1.8). The positive-sign correction is the finding that matters and does survive; the precise strength of the linkage should be held loosely.

The fiscal-risk premium is real, and most of it is Brazil's own. The size of the current alarm is closer to the raw numbers than an overcorrected control would suggest — but the correction itself is the more important result.

SYNTHESIS

The theoretical case for a fiscal risk premium in Brazil's real curve now has a number attached to it, properly controlled for global co-movement using a differenced specification rather than a levels regression that would have overstated the US's role. Brazil carries a real, substantial, mostly idiosyncratic risk premium at both 5Y and 10Y, currently running somewhat below — not dramatically below — its 2015-16 crisis level. The inflation side of the story, once properly estimated with a joint nominal-real model, is coherent on its own terms (small and positive today, spiking with genuine Brazil-specific stress, collapsing during global deflation scares) and moves with, not against, the US premium, though that link is weaker than first estimated. Two threads remain open: extending the US real curve construction to the 20Y point, and sourcing a genuine liquidity proxy for the Brazilian nominal leg to replace the current curve-fitting-error stand-in.